

Electromagnetic fields and transport coefficients in a hot pion gas

A.Gómez Nicola^{a *}, D.Fernández-Fraile^a

^aDepartamentos de Física Teórica I, II, Universidad Complutense, 28040 Madrid, Spain

We present recent results on finite temperature electromagnetic form factors and the electrical conductivity in a pion gas. The standard Chiral Perturbation Theory power counting needs to be modified for transport coefficients. We pay special attention to unitarity and to possible applications for dilepton and photon production.

1. Introduction

The most relevant properties of electromagnetic interactions in a hot and dense medium can be inferred from the retarded current-current correlator $\Pi_{\mu\nu}^R(\omega, \vec{q})$. On the one hand, Linear Response Theory provides the reaction of the system to soft external fields [1]. Thus, the electrical conductivity measures the response to a constant electric field, whereas the Debye mass parametrizes the screening of a single charge placed at the origin:

$$\sigma(T) = \lim_{\omega \rightarrow 0^+} \lim_{|\vec{q}| \rightarrow 0^+} \frac{\text{Im}(\Pi^R)_i(\omega, |\vec{q}|)}{3\omega} \quad ; \quad m_D^2 = - \lim_{|\vec{q}| \rightarrow 0^+} \lim_{\omega \rightarrow 0^+} \Pi_{00}^R(\omega, |\vec{q}|) \quad (1)$$

On the other hand, $\text{Im}(\Pi^R)_\mu^\mu(\omega, |\vec{q}|)$ is directly related to the photon yield emanated from a Relativistic Heavy Ion Collision for $q^2 = 0$ and to the dilepton rate for $q^2 = M^2$ with M the dilepton invariant mass [2].

Here we are interested in a pion gas at finite temperature and zero baryon density. Below the chiral phase transition, the dynamics of such a system can be described with Chiral Perturbation Theory (ChPT) [3], the most general low-energy expansion compatible with the spontaneous breakdown of chiral symmetry. The lagrangian is written as an expansion in pion field derivatives and masses and Weinberg's chiral power counting [4] establishes that the perturbative contribution of a given diagram is of order $(E/\Lambda_\chi)^D$ ($\mathcal{O}(E^D)$ for short) with $D = 2(N_L + 1) + \sum_d (d - 2)N_d$, N_L the number of loops, N_d the number of vertices coming from the d -derivatives lagrangian, E a typical pion energy and $\Lambda_\chi \simeq 1$ GeV. Diagrams with photon lines can be included by counting $e\Lambda_\chi = \mathcal{O}(E)$ and temperature corrections are perturbative for T well below $T_c \simeq 180$ -200 MeV.

2. Pion EM form factors and charge distribution

Only with pion degrees of freedom, the imaginary part of the current-current correlator entering the dilepton rate is directly related, to lowest order in e , to the modulus of the

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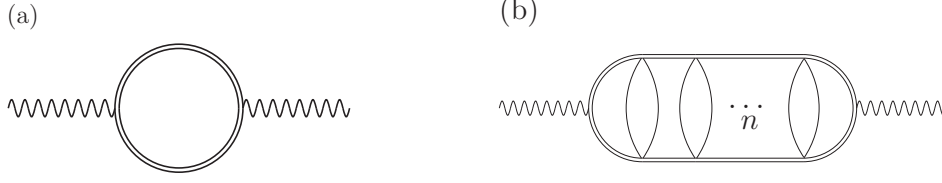


Figure 1. Dominant diagrams for transport coefficients (a) to one loop, (b) of ladder type.

pion electromagnetic form factor squared. Physically, this gives the contribution of the annihilation of two charged pions to form a dilepton pair, which is dominant for the low energy part of the spectrum.

The EM form factor at finite temperature has been calculated in [5] in ChPT to one loop. At zero energy, it provides the spatial Fourier transform of the pion charge distribution $F(|\vec{q}|^2) = Q_T(1 - \langle r^2 \rangle_T |\vec{q}|^2/6 + \dots)$, where we get for the net pion charge:

$$Q_T = 1 - \frac{1}{2\pi^2 f_\pi^2} \int_{m_\pi}^{\infty} dE \frac{2E^2 - m_\pi^2}{\sqrt{E^2 - m_\pi^2}} n_B(E; T) = 1 - \frac{m_D^2(T)}{2e^2 f_\pi^2} \quad (2)$$

where the pion decay constant $f_\pi \simeq 93$ MeV, $n_B(E; T) = [\exp(E/T) - 1]^{-1}$ and $m_D^2(T)$ is calculated to the same order [6]. Therefore, the pion charge is screened in the thermal bath proportionally to the Debye mass. As a consequence, the pion charge radius $\langle r^2 \rangle_T$ is notably increased from $T > 100$ MeV [5]. Estimating the deconfinement temperature as $(4\pi/3)\langle r^2 \rangle_{T_d}^{3/2} n_\pi(T_d) = 1$ with n_π the pion density, gives $T_d \simeq 200$ MeV, about 65 MeV lower than using the same estimate with $\langle r^2 \rangle_{T=0}$ [7].

In order to account for the ρ -resonance contribution, we have used unitarized thermal form factors F and partial waves t_{IJ} [5,8] satisfying unitarity in the center of mass frame:

$$\text{Im}F(E; T) = \sigma_T t_{11}(E; T) F^*(E; T) \quad ; \quad \text{Im}t_{11}(E; T) = \sigma_T |t_{11}(E; T)|^2 \quad (3)$$

where $\sigma_T = \sqrt{1 - 4m_\pi^2/E^2} [1 + 2n_B(E/2; T)]$ is the two-pion thermal space factor. The behaviour of $|F|^2(E; T)$ shows a clear thermal ρ broadening while reducing only slightly its mass. This is compatible with recent dilepton data in the CERN-SPS NA60 experiment, which rules out a Brown-Rho like dropping mass scenario with almost no broadening [9]. Our results are also consistent with approaches based on Vector Meson Dominance [10].

3. Transport coefficients in ChPT: the electrical conductivity

Transport coefficients are intrinsically nonperturbative. Diagrammatically, $\text{Im}\Pi^R(0^+, \vec{0})$ shows to one loop (Figure 1a) a typical “pinching pole” behaviour $G^R(\omega', |\vec{p}|)G^A(\omega', |\vec{p}|) \approx \pi\delta(\omega'^2 - E_p^2)/(2E_p\Gamma_p)$, where (ω', \vec{p}) is the four-momentum carried by the particle in the loop, $E_p^2 = |\vec{p}|^2 + m^2$ and $G^{R,A}$ are retarded/advanced propagators with the perturbatively small thermal width $\Gamma_p \ll E_p$ [11,12]. A dominant contribution proportional to the inverse width is also expected from elementary kinetic theory, since transport coefficients are proportional to mean free times. For instance, the electrical conductivity $\sigma \sim e^2 N_{ch}/(m\Gamma)$ where N_{ch} is the number of charge carriers of mass m and width Γ .

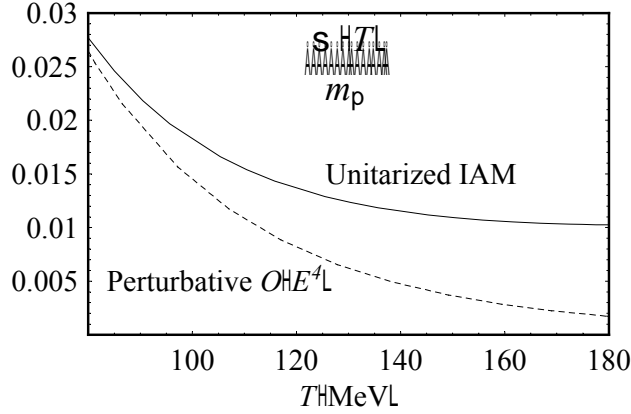


Figure 2. The electrical conductivity to leading order in ChPT, with and without unitarizing the partial waves in the thermal pion width.

In ChPT, the pion width is $\Gamma_p = \mathcal{O}(E^5)$ [13], so that the above considerations lead to a redefinition of the standard chiral power counting for the calculation of transport coefficients [14]. For that purpose, we use the double-line notation for those internal lines which share the same four-momentum when the external momentum goes to zero. Those lines carry propagators with $\Gamma_p \neq 0$ and give the dominant contributions mentioned above. Therefore, double lines do not count as a chiral loop suppression in this new chiral power counting. Instead, they are assigned a nonperturbative factor Y , which we estimate by calculating the leading order diagram in Figure 1a. Thus, we write the contribution of that diagram to the electrical conductivity as $\sigma^{(0)} = e^2 m_\pi Y$. In the standard chiral power counting we would have $Y = \mathcal{O}(E^2)$. The next step is to identify the dominant diagrams with this new counting. As it happens in simple scalar theories [11], those are ladder diagrams of the type showed in Figure 1b, which in our case are $\mathcal{O}(E^{2n} Y^{n+1})$ with n the number of rungs. Note that the loop rungs carry single lines.

In order to understand the behaviour of the relevant diagrams in ChPT, let us consider two different regimes: first, for $T \ll m_\pi$ the typical spatial loop momenta $p = \mathcal{O}(\sqrt{m_\pi T})$ and $E \simeq m_\pi$, we find $Y \simeq \sqrt{m_\pi/T}$, so that ladder diagrams could be increasingly important. However, to the leading $1/\Gamma$ order, only the spectral functions of the rung loops contribute [12] and every one of those is proportional to $p_{CM}/m_\pi \sim \sqrt{T/m_\pi} = 1/Y$ where p_{CM} is the center of mass momentum for $\pi\pi$ scattering of the double-pion (on-shell) lines attached to the rung loop. Therefore, $\sigma \propto \sqrt{m_\pi/T}$ for very low T and ladder diagrams give only perturbative corrections to the proportionality constant. The second regime of interest is $T \sim m_\pi$, where $p = \mathcal{O}(T)$ and $E \propto T$. Here, unitarization effects have to be taken into account in the partial waves entering the pion width. Qualitatively, unitarity makes the conductivity change its decreasing behaviour with T , as showed in Figure 2. Note that this change of behaviour occurs near T_c , which is reasonable since σ grows with T in the QGP phase [15]. A similar behaviour is expected for other transport coefficients [16]. In addition, although ladder diagrams still remain formally perturbative in ChPT, the presence of derivative vertices increasing with T makes them numerically

important near T_c , where the corresponding Boltzmann-like integral equations for the effective vertices [12] have to be solved and so we will do elsewhere.

Finally, let us comment on a phenomenological application of our results regarding the photon spectrum. From the definition of the conductivity in (1), the equilibrium photon rate at vanishing energy is proportional to $T\sigma(T)$ [14]. This translates directly into a prediction for the photon yield at vanishing transverse momentum, where the hadron gas dominates over the QGP. Using a simple cylindrical Bjorken expansion with parameters typical of the CERN WA98 experiment [17] gives $\omega dN_\gamma/d^3\vec{q}(q_T \rightarrow 0^+) \simeq 5.6 \times 10^2 \text{ GeV}^{-2}$. This value is reasonably close to a linear extrapolation of the two closest experimental points to the origin in [17] and is also compatible with recent theoretical analysis [18]. Our result highlights the importance of considering a nonzero pion width and resonances in $\pi\pi$ scattering for the photon spectrum near zero energy.

4. Conclusions

We have presented a recent analysis of electromagnetic properties of a pion gas. While the pion electromagnetic form factor can be computed and unitarized in standard ChPT, the calculation of transport coefficients requires a redefinition of the standard chiral power counting in order to account for typical contributions proportional to the inverse particle width. Our results show phenomenological and theoretical consistency in the context of dilepton and photon production at low energies and open up the possibility of studying in ChPT other physically relevant transport coefficients such as viscosities.

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